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The onset of three-dimensionality and time-dependence in Görtler vortices: neutrally stable wavy modes

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Recently Hall & Seddougui (1989) considered the secondary instability of large-amplitude Görtler vortices in a growing boundary layer into a three-dimensional flow with wavy vortex boundaries. They obtained a pair of coupled, linear ordinary differential equations for this instability which constituted an eigenproblem for the wavelength and frequency of this wavy mode. In the course of investigating the nonlinear version of this problem (Seddougui & Bassom 1990), we have found that the numerical work of Hall & Seddougui (1989) is incomplete; this deficiency is rectified here. In particular, we find that many neutrally stable modes are possible; we derive the properties of such modes in a high-wavenumber limit and show that the combination of the results of Hall & Seddougui and our modifications lead to conclusions which are consistent with the available experimental observations.

1. Introduction

The purpose of this article is to repeat and improve the calculation of Hall & Seddougui (1989, hereinafter referred to as HS), who were concerned with obtaining an asymptotic description of the three-dimensional breakdown of steady, spanwise periodic Görtler vortices. These authors noted that in the experiments of Bippes & Görtler (1972) and of Aihara & Koyama (1981) this breakdown led to a time–periodic flow with wavy vortex boundaries similar to those that occur in the Taylor problem. In order to investigate this phenomenon theoretically, HS superimposed small spanwise periodic travelling waves on the Görtler vortices and monitored their development.

The first analytical work concerned with Görtler vortices concentrated on the linear stability of external flows over concave walls. However, Hall (1982*a*, *b*, 1983) showed that much of this early work was fundamentally flawed for it invoked the parallel flow approximation; but Hall demonstrated that this assumption is unjustifiable except in the limit of small vortex wavelength. Moreover, in this limit, the Görtler instability may be described by an asymptotic structure which accounts for boundary-layer growth in a rational manner. This asymptotic structure was obtained by Hall (1982*a*) for the case of infinitesimal-amplitude vortices and this was used by Hall (1982*b*) to determine the modified structure in the case of weakly nonlinear vortices. Subsequently, Hall & Lakin (1988) used this latter work to

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deduce the flow configuration for fully nonlinear, high-wavenumber vortices, at which point the mean flow correction generated by the presence of the vortices becomes as large as the basic (undisturbed) flow itself. These fully nonlinear vortices are of the type whose stability to travelling wave disturbances was considered by HS. Hall & Lakin (1988) demonstrated that for these large-amplitude vortices the flow structure consists of essentially three distinct regions. The main vortex activity is restricted to a central 'core' region which is bounded by two thin shear layers. The vortices decay exponentially within these shear regions and outside these zones the mean flow is governed by the usual boundary-layer equations.

HS imposed infinitesimal secondary instabilities upon the flow within the shear layers; these modes took the form of short-wavelength, high-frequency travelling waves which were $\frac{1}{2}\pi$ radians out of phase with the fundamental in the spanwise direction so that any instabilities that occurred produced locally wavy vortex boundaries in the shear layers. It was shown in HS that the governing equations for these secondary modes takes the forms

$$\frac{d^2v}{d\eta^2}-(1+\tfrac{2}{3}i\Omega)\,\eta v-\tfrac{2}{3}iKv-V^2v+\tfrac{2}{3}\surd 6\,Vw=0, \tag{1.1a}$$

$$\frac{d^2w}{d\eta^2}-i(\Omega\eta+K)\,w+2V^2w-\tfrac{1}{3}\surd 6i(\Omega\eta+K)\,Vv=0. \tag{1.1b}$$

In this pair of coupled ordinary differential equations for the functions $v(\eta)$ and $w(\eta)$, η is an $O(1)$ coordinate based upon the thickness of the shear layers, (v,w) are the normal and spanwise components of the velocity of the travelling wave disturbance, and K and Ω are the (dimensionless) wavenumber and frequency of the imposed perturbation. Further, the function $V(\eta)$ satisfies the Painlevé equation,

$$\frac{d^2V}{d\eta^2}-\eta V=V^3, \tag{1.1c}$$

with $V\sim(-\eta)^{\frac{1}{2}}$ as $\eta\to-\infty$ and $V\sim\surd 2\,\mathrm{Ai}(\eta)$ as $\eta\to\infty$, see Hastings & McLeod (1980). For a complete description of the derivation of (1.1), together with an extended account of the previous theoretical and practical work relating to Görtler vortices, see HS. However, we do note that there is a difference in the third term of (1.1*a*) and the corresponding one in HS: this is due to a typographical error in that paper.

HS concentrated on locating solutions of (1.1) for which the flow is neutrally stable, or, in other words, on finding solutions of (1.1) for which K and Ω are real. To ensure that the travelling waves were confined within the shear layers, it was necessary to impose the boundary conditions

$$v,w\to 0 \quad \text{as} \quad \eta\to\pm\infty. \tag{1.1d}$$

This then implies that as $\eta\to\infty$, (1.1*a*, *b*) can be written as

$$\frac{d^2v}{d\eta^2}-(1+\tfrac{2}{3}i\Omega)\,\eta v-\tfrac{2}{3}iKv=0, \tag{1.2a}$$

and

$$\frac{d^2w}{d\eta^2}-i(\Omega\eta+K)\,w=0, \tag{1.2b}$$

so that in this limit two independent solutions for v and w can be found in terms of the Airy function, Ai. For $\eta \to -\infty$, HS demonstrated that (1.1 a, b) assume the forms

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\eta^2} - \tfrac{2}{3}\mathrm{i}\Omega\eta v - \tfrac{2}{3}\mathrm{i}Kv = -\tfrac{2}{3}\surd 6(-\eta)^{\frac{1}{2}} w, \tag{1.3a}$$

$$\frac{\mathrm{d}^2 w}{\mathrm{d}\eta^2} - (2+\mathrm{i}\Omega)\,\eta w - \mathrm{i}Kw = \tfrac{1}{3}\surd 6\mathrm{i}\Omega\eta(-\eta)^{\frac{1}{2}} v + \tfrac{1}{3}\surd 6\mathrm{i}K(-\eta)^{\frac{1}{2}} v. \tag{1.3b}$$

Then the appropriate expansions within (1.3) take the form

$$v = (v_{01} + \ldots)\exp(-\phi|\eta|^{\frac{3}{2}}), \quad w = |\eta|^{\frac{1}{2}}(w_{01} + \ldots)\exp(-\phi|\eta|^{\frac{3}{2}}),$$

where
$$243\phi^4 + 36\phi^2(6+5\mathrm{i}\Omega) - 32\Omega^2 = 0,$$

and the two roots of this equation with positive real part were used in order to generate two independent solutions of (1.3) with $v, w \to 0$, $\eta \to -\infty$.

These asymptotic solutions for v and w as $|\eta| \to \infty$ were taken as initial values in the numerical integration scheme used to solve (1.1). These equations were written as a system of four first-order differential equations which was solved using a standard fourth-order Runge–Kutta method. The integration procedure was started at $\eta = -\infty$ and at $\eta = \infty$ and continued to $\eta = 0$, finding two independent solutions from each direction. At $\eta = 0$ the continuity of a linear combination of the independent solutions from each direction produced a problem of the form

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}, \tag{1.4}$$

where $\boldsymbol{A}(K, \Omega)$ is a 4×4 complex valued matrix and $\boldsymbol{x}$ is a vector containing the coefficients of the independent solutions from $\eta = \pm\infty$. Clearly there is only a non-trivial solution of (1.4) if $\det(\boldsymbol{A}) = 0$. HS found real values of K and Ω for which det ($\boldsymbol{A}$) = 0 (and hence for which there are non-trivial neutrally stable travelling waves) by employing a Newton–Raphson iteration scheme for two variables.

HS focused their attention on the region $K, \Omega > 0$ and found only one eigenvalue pair, which was located at

$$(K, \Omega) = (4.156, 0.742). \tag{1.5}$$

They speculated that other eigenpairs might possibly exist at higher values of K and Ω, although none were found. Additionally, HS compared these theoretical findings with the results of the experimental observations by Kohama (1988) and Peerhossaini & Wesfreid (1988 a). They reported good qualitative agreement with these practical results, although the lack of details given in these papers concerning the specific experimental configurations used prevented a detailed quantitative comparison being made.

Recent work by Seddougui & Bassom (1990) has been chiefly concerned with the extension of HS to the nonlinear regime. Seddougui & Bassom have shown that at the point at which the secondary wavy mode becomes nonlinear, the steady vortex flow of HS is affected by self-interactions of the wavy mode and the problem is then governed by an infinity of coupled second-order ordinary differential equations. In the course of this work we reconsidered the linear problem of HS and discovered that the numerical results quoted in that paper are incomplete. In particular, we found eigenvalue pairs lower than (1.5), have shown that there are (plausibly) an infinite number of real-valued solution pairs of (1.1), and have obtained an asymptotic description of the solution of this equation for $K \gg 1$.

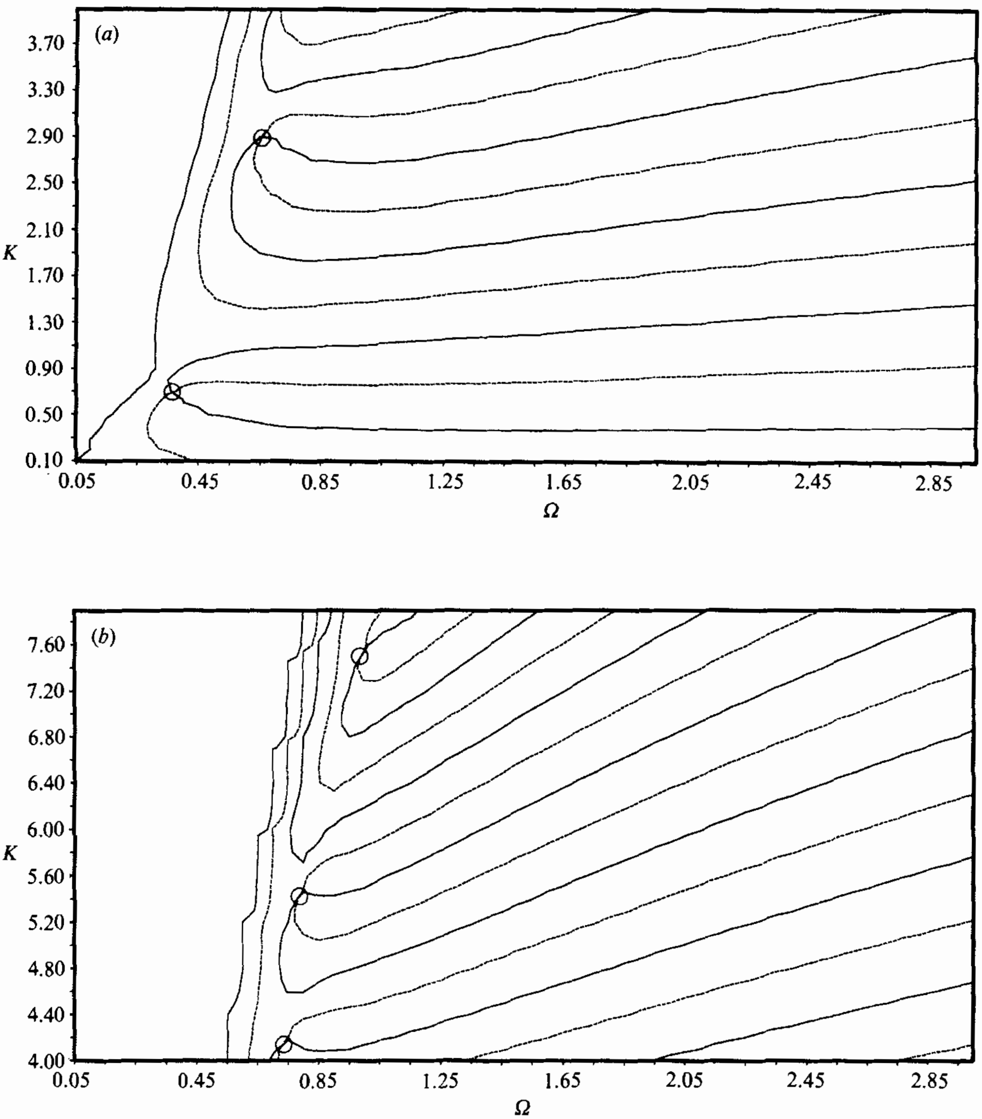

FIGURE 1. Sketches of the contours in (K, Ω)-space on which the real and imaginary parts of det $(\boldsymbol{A})$, defined by (1.4), vanish. On solid lines $\mathrm{Re}\,(\det \boldsymbol{A}) = 0$, on broken lines $\mathrm{Im}\,(\det \boldsymbol{A}) = 0$. We have a solution for neutral wavy modes wherever these contours meet and five such locations are ringed, see (2.1). Here $0.05 \leqslant \Omega \leqslant 3$, and (a) $0.1 \leqslant K \leqslant 4$, (b) $4 \leqslant K \leqslant 7.9$.

The procedure for the remainder of this paper is as follows. In §2 we present a revised solution of (1.1) for $O(1)$ values of K and Ω, in §3 we consider the case $K \gg 1$ and finally we draw some conclusions.

2. The numerical solution of (1.1)–(1.3)

To obtain real eigenvalues of the system (1.1)–(1.3) we employed the numerical method briefly described in the previous section with one modification. Instead of using the double Newton–Raphson iteration scheme to locate the neutral modes, we

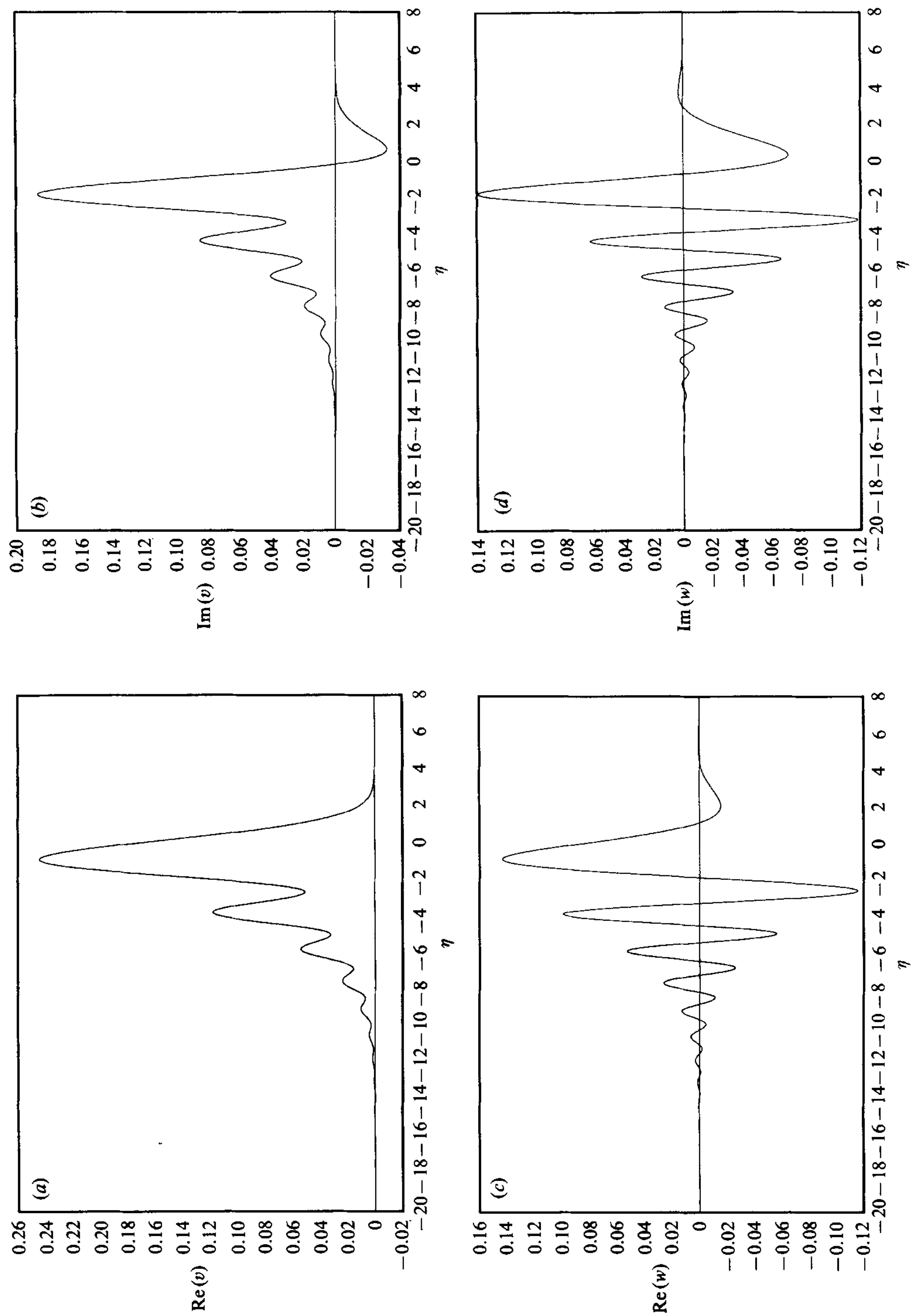

FIGURE 2. Neutral eigenfunctions $v(\eta)$ and $w(\eta)$ for $(K, \Omega) = (0.690, 0.372)$. (*a*) Re$(v)$, (*b*) Im$(v)$, (*c*) Re$(w)$, (*d*) Im$(w)$.

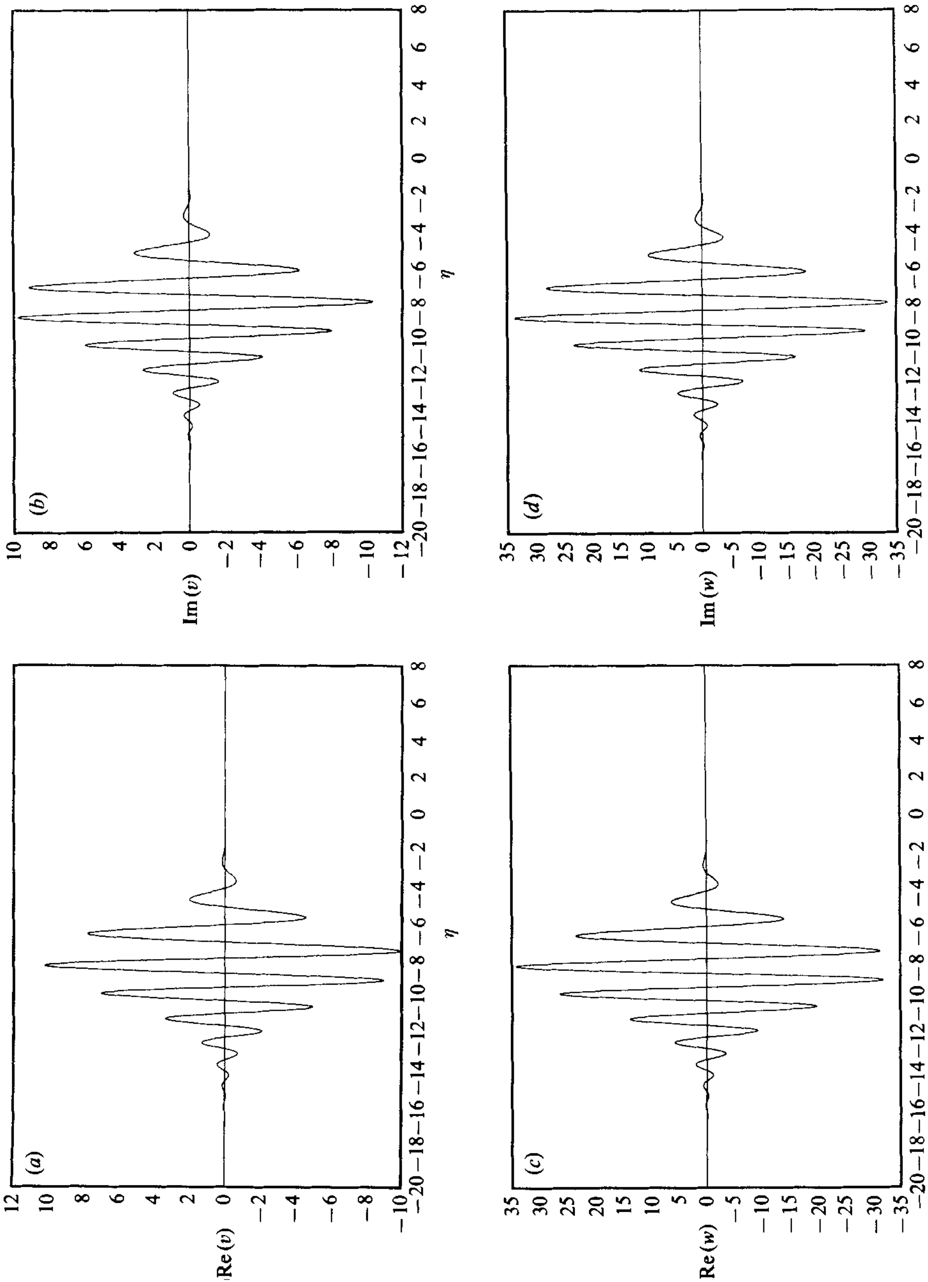

FIGURE 3. Neutral eigenfunctions $v(\eta)$ and $w(\eta)$ for $(K, \Omega) = (9.60, 1.17)$. (*a*) Re (v), (*b*) Im (v), (*c*) Re (w), (*d*) Im (w).

considered a whole range of real pairs (K, Ω) and for each chosen pair we computed the matrix $\boldsymbol{A}$ defined by (1.4). Then, using a standard package, we constructed the contours in the (K, Ω)-plane on which $\mathrm{Re}\,(\det \boldsymbol{A}) = 0$ or $\mathrm{Im}\,(\det \boldsymbol{A}) = 0$. These lines, for the region $0.05 \leqslant \Omega \leqslant 3$, $0.1 \leqslant K \leqslant 7.9$ are illustrated in figure 1, where solid lines denote contours on which $\mathrm{Re}\,(\det \boldsymbol{A}) = 0$ and broken lines those on which $\mathrm{Im}\,(\det \boldsymbol{A}) = 0$. Plainly, where these contours cross we have a solution for infinitesimal, neutrally stable wavy modes. We have omitted the contours on the left-hand sides of the figures. The absolute values of the determinant of $\boldsymbol{A}$ within these zones are very small and the interpolation used to construct the contours here results in an extremely congested diagram in which contours of $\mathrm{Re}\,(\det \boldsymbol{A}) = 0$ and $\mathrm{Im}\,(\det \boldsymbol{A}) = 0$ run very close together. However, careful analysis shows that although these contours are close to each other they do not intersect and so we can conclude that there are no neutral pairs (K, Ω) in these regions. Consequently, we have chosen to omit the densely packed contours here to avoid cluttering the figure with unnecessary detail.

Having identified approximate values of possible wavenumbers and frequencies for this secondary mode using this contouring technique, we then applied the Newton–Raphson iteration scheme to obtain more accurate values for these parameters. Overall, we found eight eigenpairs, namely

$$\left.\begin{array}{llll}(0.690, 0.372), & (2.900, 0.659), & (4.156, 0.742), & (5.435, 0.795), \\ (7.53, 1.00), & (9.60, 1.17), & (11.4, 1.27), & (15.7, 1.60),\end{array}\right\} \qquad (2.1)$$

of which the lowest five are marked on figure 1. We found no more eigenpairs within the size of grid used ($0 < K \leqslant 16$, $0 < \Omega \leqslant 3$), but there is no reason to doubt that if this area were suitably enlarged more neutral pairs could be identified. Importantly, we deduce from (2.1) that there are neutral pairs lower than (1.5). In a practical setting, if the frequency of the imposed perturbations on a steady vortex flow were gradually increased from zero, then the mode with the lowest-frequency eigenvalue would be expected to be the most dangerous as it would occur first. The eigenfunctions corresponding to this mode are shown in figure 2 and the asymmetry of these functions is noticeable. Also, for the case plotted here, $\mathrm{Re}\,(v) > 0$ across the whole of the dominant part of the mode, whereas the spanwise velocity component w has a much more oscillatory nature.

For the higher eigenvalues it was found that it became increasingly more difficult to locate neutral modes accurately and although we have given only eight pairs in (2.1), we believe that there is indeed an infinite sequence of neutral modes. Inspection of the eigenfunctions of figure 2, of those corresponding to the pair (4.156, 0.742) (presented in HS), and those corresponding to (9.60, 1.17), illustrated in figure 3, suggests some definite trends. In particular, the eigenfunctions are effectively confined to a region in η-space centred somewhere to the left of $\eta = 0$. Furthermore, for the higher modes, the majority of the disturbance shifts to increasingly more negative values of η, and becomes ever more symmetrical in appearance. The ratio of the spanwise and normal velocity components, $|w|/|v|$, increases. These behaviours led us to an analytical consideration of the problem (1.1)–(1.3) in the limit $K \gg 1$ in an attempt to elucidate the governing behaviour of neutral modes in this high-wavenumber limit. This work is the subject of the following section.

3. The high-wavenumber solution of (1.1)–(1.3)

Here we consider the solution of (1.1)–(1.3) for $K \gg 1$. It is found convenient to seek neutral modes in which the wavenumber K and the frequency Ω take the forms

$$K = (\beta + \beta^{\frac{2}{5}} K_1 + \ldots), \quad \Omega = \Omega_0 \beta^{\frac{3}{5}}, \tag{3.1}$$

where $\beta \gg 1$ and Ω_0, K_1 are $O(1)$ constants. Further, we suppose that the mode is concentrated within a region of thickness $O(\beta^{-\frac{1}{5}})$ centred at a point $O(\beta^{\frac{2}{5}})$ from the origin $\eta = 0$. If we then write

$$\eta + \frac{K}{\Omega} = -\beta^{\frac{2}{5}} \eta_c + \beta^{-\frac{1}{5}} y, \tag{3.2a}$$

and note that since for large negative η, $V \sim (-\eta)^{\frac{1}{2}} + \ldots$, we also have

$$V = \beta^{\frac{1}{5}} \left(\eta_c + \frac{1}{\Omega_0} \right)^{\frac{1}{2}} \left[1 - \frac{\beta^{-\frac{3}{5}} (y - K_1/\Omega_0)}{2(\eta_c + 1/\Omega_0)} + O(\beta^{-\frac{6}{5}}) \right]. \tag{3.2b}$$

These scalings suggest that

$$v = v_0 + \beta^{-\frac{3}{5}} v_1 + \ldots, \quad w = \beta^{\frac{1}{5}} (w_0 + \beta^{-\frac{3}{5}} w_1 + \ldots), \tag{3.3}$$

and inserting these expansions in (1.1) and equating coefficients of leading powers of β we find that $\eta_c = 0$. At next order we obtain the coupled equations for the functions v_0 and w_0:

$$\frac{\mathrm{d}^2 v_0}{\mathrm{d}y^2} - \tfrac{2}{3} \mathrm{i} \Omega_0 y v_0 + \frac{2\sqrt{6}}{3 \Omega_0^{\frac{1}{2}}} w_0 = 0, \tag{3.4a}$$

$$\frac{\mathrm{d}^2 w_0}{\mathrm{d}y^2} + \left(\frac{2}{\Omega_0} - \mathrm{i} \Omega_0 y \right) w_0 - \tfrac{1}{3} \mathrm{i} (6 \Omega_0)^{\frac{1}{2}} y v_0 = 0, \tag{3.4b}$$

with boundary conditions

$$v_0, w_0 \to 0 \quad \text{as} \quad y \to \pm \infty. \tag{3.4c}$$

By eliminating w_0 between (3.4*a, b*) we can derive a fourth-order differential equation for v_0 which may be solved numerically using standard Runge–Kutta ideas. However, careful inspection and analysis of (3.4) reveals that a solution of the system for which $v_0, w_0 \to 0$ as $|\eta| \to \infty$ is possible for all non-zero real Ω_0. Without loss of generality, we can find solutions of (3.4) such that $\mathrm{Re}(v_0)$ and $\mathrm{Re}(w_0)$ are even functions about $y = 0$ whilst $\mathrm{Im}(v_0)$ and $\mathrm{Im}(w_0)$ are odd functions. As Ω_0 is not determined at this stage, we are forced to consider the next-order equations. These are found to be

$$\frac{\mathrm{d}^2 v_1}{\mathrm{d}y^2} - \tfrac{2}{3} \mathrm{i} \Omega_0 y v_1 + \frac{2\sqrt{6}}{3 \Omega_0^{\frac{1}{2}}} w_1 = \tfrac{1}{3} (6 \Omega_0)^{\frac{1}{2}} \left(y - \frac{K_1}{\Omega_0} \right) w_0, \tag{3.5a}$$

$$\frac{\mathrm{d}^2 w_1}{\mathrm{d}y^2} + \left(\frac{2}{\Omega_0} - \mathrm{i} \Omega_0 y \right) w_1 - \tfrac{1}{3} \mathrm{i} (6 \Omega_0)^{\frac{1}{2}} y v_1 = \left(2 w_0 - \frac{\mathrm{i} \Omega_0^{\frac{3}{2}} y v_0}{\sqrt{6}} \right) \left(y - \frac{K_1}{\Omega_0} \right), \tag{3.5b}$$

with

$$v_1, w_1 \to 0 \quad \text{as} \quad y \to \pm \infty. \tag{3.5c}$$

Now as the homogeneous forms of (3.5) are precisely those equations given in (3.4), the set (3.5) has a solution only if a certain compatibility conditions holds. To derive

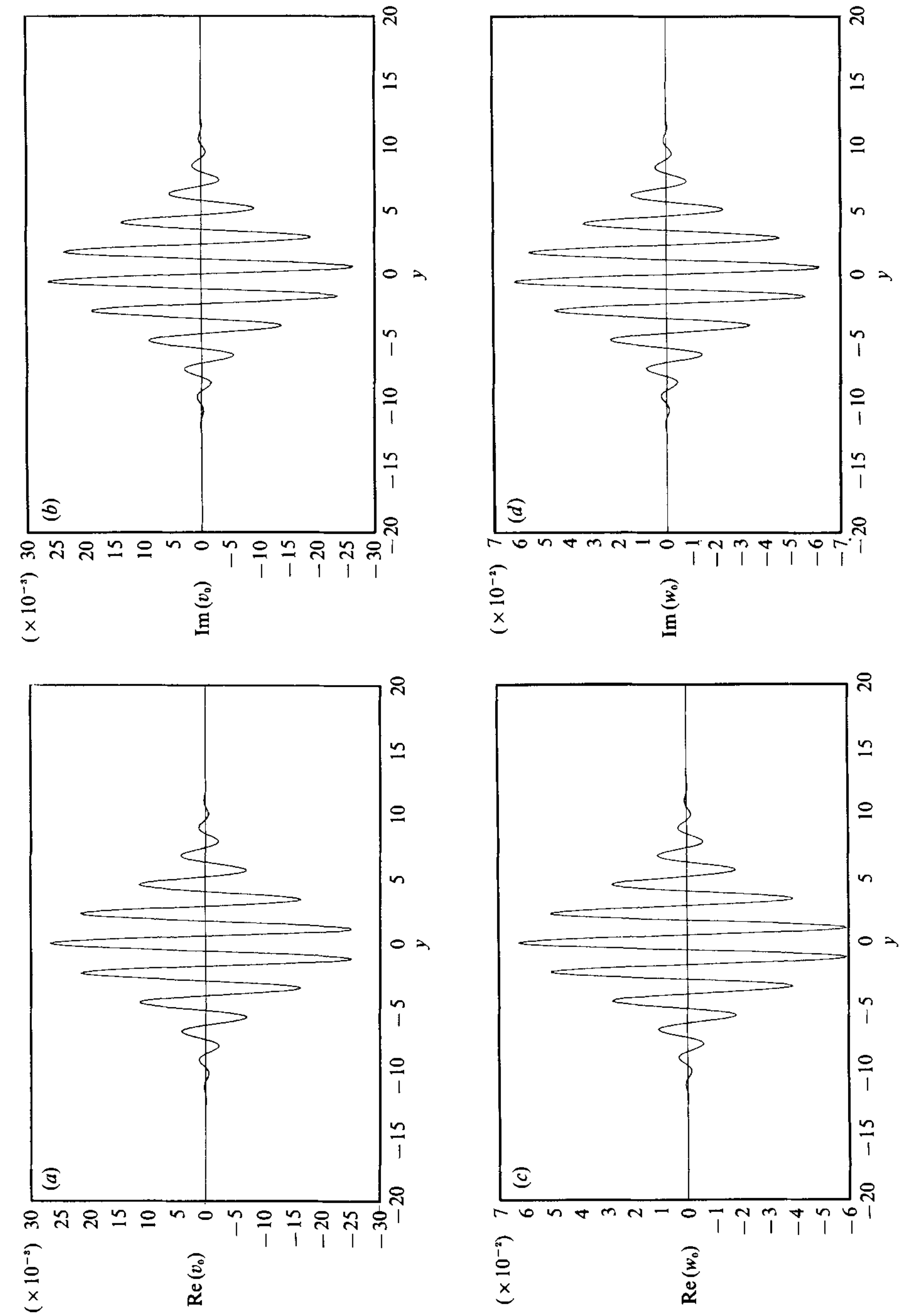

FIGURE 4. Asymptotic eigenfunctions $v_0(y)$ and $w_0(y)$ (defined by (3.3)) in the limit $K \gg 1$ with $\Omega_0 = 0.26$.

this compatibility criterion we consider the system adjoint to (3.4) which in this case is formed by the functions $(g_1(y), g_2(y))$ where

$$\frac{\mathrm{d}^2 g_1}{\mathrm{d}y^2} - \tfrac{2}{3}\mathrm{i}\Omega_0 y g_1 - \tfrac{1}{3}\mathrm{i}(6\Omega_0)^{\frac{1}{2}} y g_2 = 0, \tag{3.6a}$$

$$\frac{\mathrm{d}^2 g_2}{\mathrm{d}y^2} + \frac{2\sqrt{6}}{3\Omega_0^{\frac{1}{2}}} g_1 + \left(\frac{2}{\Omega_0} - \mathrm{i}\Omega_0 y\right) g_2 = 0, \tag{3.6b}$$

with

$$g_1, g_2 \to 0 \quad \text{as} \quad y \to \pm\infty. \tag{3.6c}$$

As before, we can obtain solutions of (3.6) in which the real parts g_1 and g_2 are even functions whilst the respective imaginary parts are odd functions. We multiply (3.5*a*) by g_1, (3.5*b*) by g_2, add the resulting equations and integrate by parts over the range $(-\infty, \infty)$. Then on comparing imaginary parts of the resulting eigenrelation we find that (3.5) has a solution only if

$$\tfrac{1}{3}(6\Omega_0)^{\frac{1}{2}} \int_0^\infty (y \operatorname{Im}(g_1 w_0))\,\mathrm{d}y + 2\int_0^\infty (y \operatorname{Im}(g_2 w_0))\,\mathrm{d}y - \frac{\Omega_0^{\frac{3}{2}}}{\sqrt{6}} \int_0^\infty (y^2 \operatorname{Re}(g_2 v_0))\,\mathrm{d}y = 0, \tag{3.7}$$

where the evenness and oddness of the various functions about $y = 0$ has been utilized. Incidentally, we note that on equating real parts of the eigenrelation we obtain $K_1 = 0$. Numerically, we found that (3.7) is satisfied for $\Omega_0 \sim 0.26$ and the respective disturbance functions v_0 and w_0 for this case are presented in figure 4.

This then suggests that high-frequency, high-wavenumber modes are indeed possible; in particular if $K \gg 1$ we have $\Omega \sim 0.26K^{\frac{3}{5}}$. Additionally, the position of the disturbance within the shear layer moves towards the core region of vortex activity, the perturbation is confined within a thin, $O(K^{-\frac{1}{5}})$ sized zone relative to the depth of the shear layer, and the spanwise velocity component of the wavy mode is $O(K^{\frac{1}{5}})$ times that of the normal component. These asymptotic results provide satisfactory agreement with the higher numerical values calculated in (2.1) and it is observed that there is resemblance between the asymptotic eigenfunctions of figure 4 and those corresponding to the eigenpair (9.60, 1.17) shown in figure 3. We also remark that here we have not specified K and have only assumed that it is some large quantity. In practice the actual possible values for $K \gg 1$ would be determined by a higher-order problem but we do not pursue such a consideration here. Instead, we feel that the principal result of this analysis is that, almost certainly, there is an infinity of neutral wavy modes for this flow structure. This confirms the conjective presented by HS, who postulated that neutral wavy modes may possibly exist at wavenumbers higher than that for the mode with eigenvalues given by (1.5).

4. Conclusions

In this article, which should be read in conjunction with HS, we have demonstrated that there are many solutions for infinitesimal, neutrally stable wavy modes confined to the shear layers of a fully nonlinear, high-wavenumber Görtler vortex. This is at variance with the result of HS who found only one such mode: although their conclusion, that such a large-amplitude vortex is unstable to modes trapped within the shear layers, remains unaltered. Our results are an improvement on those of HS due to the use of the contouring method described in §2. This ensured that all possible neutral mode solutions within the region $0 < K \leqslant 16$, $0 < \Omega \leqslant 3$ should be identified. This is in contrast to the scheme employed by HS, who used the Newton–Raphson technique alone and relied on finding all possible neutral solutions by using a sufficiently wide variety of initial estimates to which the iteration procedure was applied.

Since the wavy modes are stable when their non-dimensional frequency $\Omega = 0$, HS noted that as Ω is increased the mode with the lowest frequency is potentially more dangerous than the other modes as it occurs first. However, at large values of Ω it is unclear which is the most important mode as we would need to identify the mode with the largest growth rate. The contour plot in figure 1 shows that $(K, \Omega) = (0.690, 0.372)$ is the lowest mode within the region $K > 0$, $\Omega > 0$ and that, plausibly, an indefinite number of neutral modes exists. In particular there are modes with $K \gg \Omega \gg 1$, whose asymptotic structure has been discussed in §3.

The physical and experimental implications of our revised analysis follow similar lines to those given in HS and to which the reader is referred. We remark that the wavy mode described here can be closely related to the practical observations of Peerhossaini & Wesfreid (1988*a*, *b*). Indeed, these authors show that the lateral oscillations of the vortices are the result of 'interaction between a transverse travelling wave and the Görtler vortices, reminiscent of the wavy vortex motion in Taylor–Couette instability'. Thus wavy vortices have been seen in practice and Peerhossaini & Wesfreid (1988*b*) commented on two particular forms which they called the 'oscillatory mode' and 'jump and stay' motions. In the former case, Peerhossaini & Wesfreid observed that as the Görtler number was increased the upwash plane between successive Görtler vortices began to oscillate around its stationary position. The spectrum of the frequencies present within the oscillation contained one distinctive peak and we interpret this event as marking the onset of the secondary instability described both here and in HS.

At larger amplitudes of the oscillatory mode, the observed motion consisted of a series of jumps from one state into another with a long residence time at each stage – the 'jump and stay' motion. The theoretical explanation of this phenomenon is provided by our finding that, on the basis of linear theory, there are many possible neutral pairs (K, Ω). If, according to a weakly nonlinear theory (work described in Seddougui & Bassom 1990), these neutral states prove to be supercritically stable then this would imply that stable finite-amplitude oscillatory modes with a multitude of possible wavenumber–frequency pairings (K, Ω) may be achieved. As the Görtler number increases it is probable that the relative importance of each of these states changes and thence the development of the flow would consist of the 'jump and stay' motion described by Peerhossaini & Wesfreid. As the flow evolved the oscillatory mode would jump from state to state as the global significance of each of the configurations changed.

In summary, we feel that our principal finding here is that the existence of many possible neutral linear modes is predicted. A natural question then concerns which of these modes is likely to be the most importance in practice? This may be resolved by pursuing weakly nonlinear and, ultimately, fully nonlinear analyses of these wavy modes – a task currently being tackled by Seddougui & Bassom. When concrete conclusions become available from this study, the full implications of our work for this wavy vortex problem should become clearer.

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